ALFRÉD RÉNYI INSTITUTE OF MATHEMATICS
1053 Budapest, Reáltanoda u. 13-15; 1364 Budapest, Pf. 127
Phone: +36 1 483 8302; Fax: +36 1 483 8333
e-mail: palfy.peter.pal@renyi.mta.hu; web page: http://www.renyi.mta.hu

I. Main duties of the research unit in 2012

The fundamental goal of the Alfréd Rényi Institute of Mathematics is to pursue research of high international standing in pure mathematics. The institute is an important center of mathematics internationally. In 2012 it further improved its reputation both in Hungary and abroad, thanks to the outstanding achievements of its researchers. The Abel-prize, the greatest international prize in mathematics was awarded to a research professor emeritus of the institute. A new five-year research project supported by the European Research Council's Advanced Grant was launched, and the decision of supporting another project was announced in 2012. With these the institute becomes the host institution of four Advanced Grants. The international prestige of the institute was also witnessed by the fact that the Rényi Institute was chosen to host the annual meeting of the consultative body of the European mathematical institutes (ERCOM) in 2012. It should also be mentioned that the American Institute of Mathematics has selected the Rényi Institute for the venue of one of its conferences.

The scientific tasks of the institute concentrate on fundamental research. However, significant efforts were devoted to some topics of applied mathematics as well. The main applied areas investigated in the institute are bioinformatics and cryptography, but applications of mathematical statistics are also relevant. The institute is organized in the framework of nine scientific departments. The three research groups supported by the Academy's Momentum programme (Cryptography since 2009, Low Dimensional Topology since 2010, Group Theory since 2012), as well as the research groups with ERC grants (Number Theory since 2008, Geometry since 2010, Topology since 2012) work on their own independent research projects within the relevant scientific departments. The research topics are continuously adjusted to the most recent development of mathematics.

II. Outstanding research and other results in 2012

a) Outstanding research and other results

Department of Algebra

- The notion of an invariant random subgroup has been introduced. This notion aroused vivid interest of many researchers worldwide, and a workshop on this subject has been organized in Israel.
- The distinguished role of the Gauss Lemma in valuation theory has been pointed out. This observation yields a short proof of the well-known Jaffard-Ohm theorem, which states that every lattice-ordered abelian group arises as the divisibility theory of a suitable Bezout ring.
- The group of automorphisms of strongly upper triangular matrix rings has been described in terms of the groups of automorphisms of the subrings and bimodules in them. This has made it possible to carry over the theory of idempotents of semi-perfect rings to these rings in a satisfactory manner (in the framework of the bilateral – TéT – agreement between Hungary and South Africa).

- The Noether number concerning degree bounds for polynomial invariants of finite groups coincides with the Davenport constant in the case of an abelian group. Results from additive combinatorics have been generalized for the generalized Noether number, introduced by analogy to the generalized Davenport constant. For example, a lower bound has been given for the generalized Noether number of a direct product in terms of the corresponding quantities of the direct factors. One of the conclusions is that the exponent of an abelian group corresponds in the noncommutative case to a quantity that plays an essential role in the constructive invariant theory of reductive algebraic groups.
- An invariant theoretic characterization of subdiscriminants of matrices has been found and applied to the study of square-sum of representation of subdiscriminants of real symmetric matrices.
- Important results have been obtained on the asymptotic behaviour of Betti numbers, twisted torsion and other spectral invariants of sequences of semisimple locally symmetric spaces.
- Concerning the extraspecial case of the so-called non-coprime k(GV) problem, stronger upper bounds were given for the number of conjugacy classes k(GV) of the semidirect product GV where V is a completely reducible finite faithful G-module for a finite group G.
- For a non-cyclic finite group G, $\gamma(G)$ denotes the smallest number of conjugacy classes of proper subgroups of G needed to cover G. Explicit bounds and formulas were given for $\gamma(G)$ in case G is a finite linear group. In particular, it was shown that this invariant is linear in the dimension of G.
- Generalizing the notion of semi-exactness defined by Grandis, a new notion of exactness in non-pointed categories has been introduced which allows a simple categorical treatment of Kurosh-Amitsur radical theory. This approach reveals how categorical closure operators arise as radical theories, since the classes of dense and closed morphisms with respect to a closure operator turn into a corresponding pair of a radical and a semisimple class.
- Commutative orders in semigroups have been investigated from a novel point of view and have been characterized by means of the semilattice decompositions of the subsemigroup of square-cancellable elements in them. It has been shown that every semigroup of quotients of a commutative order arises as a homomorphic image of well-determined tensor product semigroup.
- For several natural classes of groups it has been shown that they contain no endoprimal group (in the framework of the agreement of Estonian and the Hungarian Academies of Sciences).

Department of Algebraic Geometry and Differential Topology

- They proved that the classical Durfee conjecture (about the signature of complete intersection surface singularities, a 40 years old conjecture) is not true. They proposed a new inequality and verified it in the homogeneous case.
- They discovered that the semicontinuity of the Hodge spectrum can be proved topologically. They analysed both the case of plane curves and the general situation.
- They established important Morse theoretical properties of manifolds with boundary. Based on these results and other algebraic surgery methods they proved the existence and key properties of Seifert forms.
- They proved the reduction theorem for Lattice Cohomology, which is an important development in the structural description of these homological groups.
- They search for certain 3-manifolds with special properties which can be realized as the boundary of the Milnor fiber of a non-isolated singularity.

- They developed a beautiful application of the Ehrhart theory, which helped to understand and calculate the Seiberg-Witten invariants of the link of a normal surface singularity, making connections between the various invariants.
- They analyzed the structure of maps between low dimensional manifolds via their singular set.
- They have proven duality theorems for the Galois cohomology of tori defined over function fields of curves over a p-adic field. It turns out that, like in the classical case of number fields, a Tate-Shafarevich group may be defined which is finite and dual to the Tate-Shafarevich group of the so-called dual torus.
- They have shown that a conjecture of S. Saito and K. Sato concerning certain Bloch-Ogus type complexes and recently published in Annals of Mathematics can in fact be stated in a sharper form. Therefore the already known cases of the conjecture are true in a stronger version.
- They have found a combinatorial description of Heegaard-Floer homology groups. In particular, they achieved several results regarding lattice homology theory; e.g. they have found a spectral sequence converging from lattice homology to Heegaard-Floer theory. Since the spectral sequence collapses in some cases, they obtained isomorphisms of these theories in such cases.
- They also showed that replacing the symplectic resolution of a surface singularity with one of its smoothings is a symplectic operation.
- They proved results significantly extending a theorem of Shalev concerning expansion in finite simple groups.
- They found a much more general version of a theorem of Lubotzky concerning product decompositions of finite simple groups, which was proved by model theory. In contrast, the new proof uses much easier group-theoretic tools.
- A surprising result, obtained a few years ago by Liebeck, MacPherson and Tent describes those primitive permutation groups where the diameters of all orbital graphs are bounded. The motivation and the tools come mainly from model theory. Using group theory it was possible to obtain partly more general, partly more precise results.

Department of Algebraic Logic

- Several first order logic axiom systems were constructed for special relativistic dynamics that allow superluminal particles. These axiom systems consist of few simple, transparent and geometrical axioms. Each of them predicts that the relativistic mass and the momenta of the superluminal particles decrease with their speeds. An extension of the axiom systems is given such that the existence of superluminal particles is logically independent of the new axiom system, the new axiom system together with the statement that superluminal particles exist forms a complete theory, and the same holds for the statement saying that there are no superluminal particles. This situation is completely analogous to the fact that Euclid's postulate of parallels is logically independent of the rest of the axioms of Euclidean geometry.
- They showed that if there are faster than light signals, then it is possible to design a relativistic hypercomputer (a computer capable to decide some non-Turing decidable problem, e.g., the consistency of ZFC set theory) even in special relativity. Moreover, they also showed that relativistic hypercomputation is possible in special relativity only if there are faster than light signals.

– It is customary to state the existence of square roots of positive quantities in the number fields used in special relativistic axiom systems, because the formula of the Minkowski metric contains the operation of taking square roots. They constructed a model of special relativity, for each dimension greater than 2, over the field of rational numbers such that inertial observers can move approximately with any speed less than that of light. This result is surprising because there is no model of special relativity in dimension 3 over the field of rational numbers in which observers can move with any speed less than that of light.
– They proved, using a field for coordinatization where not all positive members had square roots, that in 4 dimensions one can construct special relativistic coordinate systems for all observers moving with speeds less than that of light. This is solution to a problem published in 2002. The answer is surprising because it shows that this important spacetime theoretic question has different answers in three and in four dimensions.
– They proved that uniformly accelerated observers do not have special relativistic models over the field of algebraic real numbers. This result shows that, while the first-order logic theory of the real numbers suffices for investigating accelerated observers in general, it is not enough any more if we assume the existence of uniformly accelerated observers. In the latter case we have to amend the axiomatic theory with a kind of (weak) set theory.
– Adding the residuals of composition (consecutive execution of programs) to the operations of Kleene algebras, subject of a central study in computer science, is customary in the literature because they make the theory better behaved. They proved that the *-free equations valid in relational Kleene algebras expanded with the residuals of composition cannot be finitely axiomatized even using * , where * denotes the operation of iteration of programs. This result shows that the price of the above better behavior is that a good axiomatisability property of the Kleene algebras get lost.

Department of Analysis

– They described the possible dimensions of typical continuous functions (in the sense of Baire category) mapping from a compact metric space to the n-dimensional Euclidean space.
– Earlier they proved that the so-called topological Hausdorff-dimension is the appropriate tool for determining the Hausdorff-dimension of the level set for typical one-variable continuous functions defined on a compact metric space. Now they succeeded to generalize the result for functions with several variables by combining topological and metrical methods.
– Theorems on level sets of prevalent continuous functions were also obtained.
– The manuscript of the book “Introduction to Matrix Analysis and Applications” has been finished.
– In matrix analysis the matrix variance can be defined based on classical probability theory. However, some properties surprisingly differ from the classical case. The researchers succeeded in proving decomposition results, that is the variance of one or two matrices can be determined as convex combination of the variances corresponding to the appropriate projections.
– Regarding the problem whether the plane can be covered by finitely many rotated copies of a narrow vertical strip, they were able to prove that it is possible if the half-width of the strips is larger than 1/5.
– They have generalized the Wiener-Ikehara theorem using weaker assumptions than monotonicity, still obtaining an effective error term.

– Investigating approximation with so-called “truncated” polynomials with several variables, an estimation for the magnitude of convergence was obtained using Bernstein-polynomials and the modulus of continuity.

Department of Discrete Mathematics

– The study of large graphs and networks was established in the last decade. It is based on the possible definitions of convergency of graph sequences. In this respect, they achieved some progress in connection with the so-called right hand side convergency, where this new concept was defined by the number of homomorphisms of the big graph into the small graph.
– They studied what graph parameters are convergent if the graph sequence is convergent in Benjamini-Schramm sense. They extended their results on the root moments of the chromatic polynomial to a large class of graph polynomials and as a continuation; they described the matchings of Benjamini-Schramm convergent graph sequences.
– They described the limit theory of hyperfinite graph classes. It was proved that the limit objects are exactly the hyperfinite graphings. Furthermore, they proved that all the measurable parameters are estimable on these graph classes and worked out the limit theory of metric measure spaces.
– A much studied problem in the theory of graph homomorphisms is the investigation of the duality pairs. They managed to produce infinite-finite duality pairs (it was not clear if there exists any such pair) and in another paper, full characterization of them is given too. As a tool, they introduced a new concept of regular trees.
– In a paper of more than 160 pages, they proved the classical conjecture that if half of the vertices in a graph have degree k or more then the graph contains every tree of k edges as a subgraph.
– They proved a famous conjecture on linear hypergraphs. The theorem is another result of a successful application of the kernel method established earlier.
– According to a classical result, if we take a family of sufficiently many sets then there exists a subset K of given size such that every subset of K is the trace of the family. This theorem is generalized now for codes, they found the weakest condition for the existence of polynomially many codewords having all traces.
– They determined in some posets P that at most how many such copies of P can be found in the power set of the set of n elements that there are no comparable elements in two copies.
– They continued the research of traces of set families, for any $k > l$, they asymptotically determined the size of a biggest possible family that remains k-Sperner however we delete l elements of the ground set. They settled the case $l = 1$ completely including the description of the extremal families.
– The most important result in cryptography is that they found the exact optimal secret sharing schemes in the case when the access structure is defined by the edges of a tree.
– They continued the research of extremal hypergraphs: they proved the hypergraph generalization of the theorem of Erdős and Gallai about paths.
– They continued the research on the applications of the Regularity Lemma. They studied the generalizations of several classical coloring problems when the underlying graph is not complete.

- They determined the asymptotic value of the independence ratio for the direct graph product. It turned out that this graph parameter can be calculated from ratios of the size of the independent sets of the graph and the size of their neighborhoods by a simple formula. The result gives the answer for two questions of famous mathematicians. Moreover, it proves the conjecture that the value of the parameter for disjoint union of two graphs equals the maximum of the values computed for the two graphs separately.
- It was raised earlier – and solved for some particular cases – the problem of determining the maximum number of edges in a graph of order n whose edge set can be partitioned into induced subgraphs isomorphic to a given fixed graph F. The number was determined for every non-empty graph F apart from a negligible error term.
- They continued their research on realizations of degree sequences. They finished their paper on realizations and sampling of semi-regular bipartite graphs and managed to correct the well-known, but until now un-reparable error in a fundamental paper of the topic.
- In bioinformatics, they generalized some earlier results on reconstructions of DNA-code for non-binary alphabets. Furthermore, they partly determined the number of phylogenetic trees where each specialization is at least bifurcation, partly found the generator function of those phylogenetic trees, where the labels should not be unique.

Department of Geometry

- One of the basic problems in discrete geometry is to determine the most efficient packing of congruent replicas of a given convex set in the plane or in space. One of the projects surveyed the known results and obtained some new results on packings of cones and their negatives.
- A polyhedron is called triangulable if it is union of face to face tetrahedra. Five simple non-triangulable polyhedra were previously known. A new family of polyhedra was found, which could not be partitioned into tetrahedra even if face to face connection was not required. Allowing this more general type of partitioning gave rise also to new proofs for previous results.
- As a first step towards a logarithmic Brunn-Minkowski theory the logarithmic Minkowski problem was solved, and some particular cases of the logarithmic Brunn-Minkowski conjecture were verified.
- An upper bound was given for the length of the shortest path avoiding the members of a packing of balls of bounded radii connecting two points lying outside the balls at distance d from each other.
- Dowker's approximation theorems were generalized for the case when sets that are the intersection of circles of radius r are approximated by sets which are the intersection of n circles of radius r.
- They solved an important problem originating in graph drawing. Among other results, it was shown that there exist n opaque unit disks in the plane such that in whatever order we place them, the total length of the portions of their boundaries visible from above is o(n).
- The best known estimate related to the following old problem of Erdős was improved. What is the largest number f(n) with the property that every convex n-gon in the plane has a vertex v such that the number of distinct distances from v to all other vertices is at least f(n). Erdős conjectured that the answer is roughly n/2; a lower bound larger than (13/36)n was proved.
- It was shown that for any pair of positive integers k and d, there exists c = c(k,d) such that every finite point set X in d-dimensional space induces at most c regular full-dimensional simplices, whose side length is one of the k largest interpoint distances in X.

- According to Ramsey's theorem, for any positive integers k and n, there is a finite number R = R(k,n) satisfying the following condition. No matter how we color all k-element subsets of an R-element set X with two colors, X has an R-element subset, all of whose k-element subsets are of the same color. Estimating the smallest such value of R is a classic theme in combinatorics. They studied the question how the above estimates change if one restricts our attention to colorings in which a color of a k-tuple T depends on a bounded number of real parameters associated with T. It turned out that in this special case, particularly important in geometric applications, one can establish a much better upper bound, which is essentially proportional to the logarithm of the general bound.
- They investigated antipodality properties of finite sets in d dimensions. They gave sharper estimates for the maximal number of antipodal pairs among n points in the plane than former ones. They determined the minimal number of strictly antipodal pairs among n points in 4 dimensions, for any n. For d dimensions they gave estimates for the same minimal number. They gave a new proof for the maximum number of segments in an antipodal, or strictly antipodal set of segments in 3 dimensions.
- Two elements are separated by a set S, if S contains exactly one of them. A set of elements is separated by a family of sets, if any two elements are separated by at least one of the sets. They proved that any set of n points in general position in the plane can be separated by O(n log log n/ log n) convex sets, and for some point sets Ω(n/ log n) convex sets are necessary.
- A planar set S is called cover-decomposable, if there is a constant k = k(S) such that any k-fold covering of the plane with translates of S can be decomposed into two coverings. Previously, general positive results for cover-decomposability were known only for open sets. In particular, it was known that open convex polygons are cover-decomposable. A first step towards results for closed sets was made by showing that closed, centrally symmetric convex polygons are cover-decomposable. This result could be generalized for many other versions of cover decomposability.
- New special cases of the strong polarisation problem have been proved. The inverse spectral theorem was shown to be hold for an infinite class of positive semidefinite matrices. Also, it was shown that among the orthogonal cross-polytopes satisfying a norming condition, the regular one has the minimum mean width, while the at most 2-dimensional degenerate cross-polytope has the maximal mean width. Related to this result, among the n-dimensional normally distributed random vectors with a given trace of the covariance matrix the ones with maximum norm have been determined.

Department of Set Theory and General Topology

- Using the notion of topological Hausdorff dimension recently introduced by them, they gave a very precise description of the sizes of the level sets of the generic map defined on a fractal.
- They have studied the question under what condition is there a free ultrafilter on a topological space that has a base consisting only of connected sets. It is easy to see that there is a T_1 space on which every ultrafilter is like that. On the other hand, they proved the surprising and non-trivial fact that there can be no such ultrafilter on any Tychonov space.
- They have continued the study of resolvability properties of topological spaces. Their main result is the following significant sharpening of a theorem of O. Pavlov: If in a T_3 space the supremum of sizes of its closed discrete subspaces is less than the size of any non-empty open set, then the space is Ω -resolvable. Earlier it was not even known if a T_3 Lindelöf space is 3-resolvable if all non-empty open sets in it are uncountable.

- A topological space is called “base resolvable” if every base of it can be partitioned into two bases. They proved that every dense-in-itself T_3 Lindelöf space is base resolvable, moreover it is consistent that there is a dense-in-itself space which has a non-resolvable base consisting of clopen sets.
- They investigated the following tomography-related geometric reconstruction problem. Let us fix a family of compact subsets of a euclidean space. Certain test sets are said to reconstruct a member of this family if the Lebesgue measures of its intersections with the test sets determine this member. Surprisingly, in addition to measure theory, tools from algebraic topology, probability theory, and Fourier analysis played key roles in their results.
- A set of multivariable functions over a given domain is said to be a clone if it contains the projection functions and it is closed under composition. The investigation of clones over finite domains is a classical area. Recently, a definite progress has been made in the study of clones over infinite domains, although this is technically much harder. They proved that over a countably infinite domain one can find many pairwise different clones such that each one of them contains all unary functions on the domain.

Department of Number Theory

- Differences of primes were studied by analytic methods. It was proved that the set of integers representable as a difference of primes are quite dense even in rather short intervals; this is a step towards Polignac's conjecture that all even integers have this property. An irregularity property was established for the difference of consecutive primes by establishing that the quotient of consecutive such differences can be arbitrarily large as well as arbitrarily small. If we norm these differences by dividing the average value log p, conditionally (under a generally accepted conjecture) these quotients are dense in an interval.
- They investigated certain problems of combinatorial number theory. In connection with the sum-product problem it was proved that in a finite field every not too small set has the property that k-term products of 2-term sums cover the whole field.
- Properties of difference sets were investigated, with special emphasis on connections with positive character sums. Many seemingly disconnected problems can be formulated in this setting, like orthogonal latin squares, distance of codes, unbiased bases. A study of properties of general sets was finished, and a partial result was found about unbiased bases.

Department of Probability Theory and Statistics

- New results have been achieved about the secrecy capacity of multiple access channels, and on oblivious transfer capacity. The minimization of convex integral functionals subject to moment constraints has been investigated, and the results were applied to problems of risk estimation in mathematical finance.
- A bound was derived on the rate of decrease of relative entropy, for the case of Gibbs samplers, whose stationary distribution has a positive density in the n-dimensional Euclidean space. This bound can be considered as a logarithmic Sobolev inequality for the Markov chain defined by the Gibbs sampler, and implies a classical logarithmic Sobolev inequality for its stationary distribution. The results are dimension free. (The Gibbs sampler is a method for generating multidimensional distribution changing one coordinate in one step using the conditional distribution.)

- They continued their investigations on the so-called two-dimensional anisotropic random walk. This walk is a Markov chain with site dependent transition probabilities. This model was introduced by physicists and has many physical applications. Problems were studied also from mathematical point of view but there are a number of questions to be solved. One open problem is to give necessary and sufficient conditions for recurrence. They have given conditions under which the random walk is recurrent, resp. transient., but there are cases not covered, so the problem is still open in certain cases.
- A result from 1975 about the joint distribution of arbitrary independent identically distributed random variables with independent Gaussian variables. Now the result was extended for weakly dependent random variables, solving a long standing open problem in probability theory. A precise a.e. convergence criterion for sums of periodic function with bounded variation was given.
- Effective methods were developed in the study of important limit problems of probability theory and mathematical statistics in such cases when the classical methods do not work.
- It was shown that the HD motif in the amiloid beta fragment is under positive selection. (The HD motif consist of a histidine and asparagine acid.) The motif is not position specific, however, it can be found in significantly higher fraction of the homologous sequences than expected without selection. The motif participate in an unordered fragment, and probably participates in metal-binding.
- The swap Markov chain was proven rapidly mixing on the realizations of half regular degree sequences. A simple relationship was found between the swap distance between two graphs with the same degree sequence and the maximal circle decomposition of their symmetric difference.
- A new model was defined for the balance of mutation and selection. The heritability of isolated common congenital abnormalities was determined in the model. The fastest mixing chain was determined while the graph of allowed transitions form a cycle with a few extra edges.
- For statistical analysis of large graphs a new unified version of the block model with Rasch model was developed. An effective algorithm for model identification was introduced.

Applied research

The major part of the research carried out at the Rényi Institute is generated by questions raised by the inner development of mathematics. Nevertheless, together with the exploratory (theoretical) research which the Institute conducts these new results and other fundamental methods of mathematics are used in other areas as well. These include the use of the results of three-dimensional topology for DNS recombination questions (in the framework of one of their Momentum research groups), research in bioinformatics (research partly covered by an EU project) research in cryptology (in the framework of another Momentum research group), and the development of a new algorithm for scheduling of transportation problems. As part of the bioinformatics research Rényi Institute was a member of the ITFoM (Information Technology Future of Medicine) FET flagship project, which according to the latest news was not selected as one of the two winning FET flagship projects, and it is a member of the TÁMOP project “Phylogenetic comparison and investigation of Pathogenic capability of isolated porcine reproductive and respiratory syndrome virus (PRRSV) strains”. The end date of the “Comparative Genomics and Next Generation Sequencing” EU project has been changed to 2013 due to the delay of some other members of the consortium. The institute’s role in the project is unchanged; they continue the development of the new generation sequencing software.

The cryptology research group established as one of the first Momentum research projects of the Academy has successfully continued researching one of the key topics of cryptography, secret sharing. Based on earlier results they have pointed out the importance of measurability: the function which produces the secret from parts must be measurable, otherwise paradox systems can be built. The usual Shamir secret sharing scheme cannot be generalized for the infinite case since there is no uniform distribution on infinite fields. However, using Gauss processes they managed to give new constructions. They have shown that there is an approximately perfect secret sharing scheme for every access structure. Using topological properties they have managed to characterize all structures with this type of properties.

They have significant results in using mathematical statistics for different purposes, where members of the Institute give invaluable help in discovering the connections of the problems. These included the development of a test for forecasting genetic based oxidative stress, statistical inference of congenital heart abnormality and its impacts, evaluation of "new generation sequencing" measures carried out by the National Institute of Oncology, and statistical inference of the database of the Central Administration of National Pension Insurance.

Career advancement of researchers

In 2012 three researchers of the institute received the DSc title and one young researcher got his PhD. At the end of the year 14 members of the Academy (10 according to the statistical number of employees), 35 doctors of MTA (stat. num. 29), 29 researchers with PhD or CSc (stat. numb. 26) worked at the institute, 13 researchers have not yet obtained a degree. The institute puts great emphasis on involving young talents – working towards their PhD or just obtaining the degree – into the research work of the institute. In 2012 further six young researchers were employed in the new or vacant positions offered by the Academy. Altogether 15 young researchers worked in the institute in 2012, two others were on leave. The institute has an agreement with the Central European University. In this framework 17 doctoral students were supervised by members of the institute.

b) Dialogue between science and society

Unfortunately, most of the research topics in pure mathematics are not suitable for discussions for the general public. On the other hand, the international success of the researchers, foremost the Abel-prize awarded to the institute's researcher, has underlined the importance of the research conducted in the institute even in the media.

The researchers of the institute play an important role in popularizing mathematics, giving lectures for high school and university students. The institute regularly organizes an open house during the Festival of Hungarian Science, where high school students and their teachers can get information about the mathematics profession. Members of the institute take part in fostering mathematical talents by organizing several mathematical camps and other events for students interested in the subject.

III. A presentation of national and international relations

National relations

Researchers of the institute teach part time in many universities both in Budapest in other cities (Eötvös University, Budapest University of Technology, University of Debrecen, University of Szeged, Pannon University). They play an important role in doctoral schools an in Master programs. 18 members of the institute are core members of doctoral schools in various universities, they supervise 41 doctoral students. Especially important is the collaboration between the institute and the Department of Mathematics and its Applications of the Central European University. In 2012 the cooperation agreement between CEU and the institute was renewed. The lecturers and the supervisors of the Masters and doctoral programs of CEU mainly belong to the institute, including the new department chair and the leader of the doctoral program. During the international evaluation of the department in 2012 the pivotal role of the Rényi Institute in the doctoral program was highly appreciated. Also the large part of lecturers of the Budapest Semesters in Mathematics English language study abroad program for North American students belongs to the institute. This program helps to bring the fame of Hungarian mathematics to American universities, and serves as a role model for other international programs (e.g., AIT). For the institute the close contact with the new generation of mathematicians is of foremost importance. In this spirit 43 members of the institute (55 percent of all researchers) were active in teaching at universities.

The weekly seminars in the institute are attended regularly by researchers from other institutions, among them some people from universities outside Budapest as well. This way these seminars influence the whole mathematical scene in Hungary.

Members of the Rényi Institute traditionally take part in various Hungarian scientific committees well over proportion. In particular, the Mathematics Section of the Hungarian Academy of Sciences (MTA) and its committees, the Hungarian Research Fund (OTKA), and the János Bolyai Mathematical Society (BJMT) can be mentioned. The president of the Mathematics Section of MTA, the chairman and the secretary of the Mathematics Committee, one of the vice-chairmen and the secretary of the Bioinformatics Committee, the chairman of the board for the János Bolyai research fellowship, the chairman of the Mathematics and Natural Sciences subcommittee of the Council of the Academy's Research Units, the chairman of the external advisory board of the MTA Computer and Automation Research Institute, the president of the BJMT, the vice-president of the Hungarian Society for Bioinformatics are all researchers of the Rényi Institute.

International relations

The researchers of the institute have very extensive international relations. Among the coauthors of the members of the institute one finds mainly foreign mathematicians. Joint projects and jointly organized conferences are also typical. In 2012 twenty people from the institute were involved in organizing international conferences, some of them at more occasions. In 2012 the number of conferences that took place in the institute was less than the average. The most important among these was the conference on "Motivic Donaldson–Thomas theory and singularity theory" organized by the American Institute of Mathematics, one of the very few events this American organization supported in Europe. In Summer the CAST summer school and workshop related to the work of the Low Dimensional Topology "Momentum" group was funded by the European Science Foundation. In Fall, the "First International Conference on

Logic and Relativity" discussed exciting developments of connecting mathematical logic and relativity theory. The young researchers in the institute have organized the fourth "Emléktábla Workshop".

The visits in the framework of the bilateral exchange programs between the Hungarian Academy of Sciences and its partner institutions successfully contributed to the cooperation with foreign partners. With the help of these programs fruitful joint research projects, useful exchange of information, and conference participations were made possible.

Researchers of the institute took part in altogether 12 international scientific committees. For example, the vice-president of the European Set Theory Society is a member of the institute. Names of the institute's researchers can be found 146 times on the list of editorial board of various international journals. In 2012 the researchers gave altogether 218 talks at international meetings, many of these were given as an invited or plenary lecture.

In 2012 nine researchers spent more than half a year abroad at the following institutions: University of Chicago (USA), Auburn University (USA), University of Delaware (USA), City University of New York (USA), National Science Foundation (USA), Simon Fraser University (Canada), Kuwait University (Kuwait), École Polytechnique Fédérale de Lausanne (Switzerland).

Financed by the ERC and Momentum grants ten foreign researchers worked in the institute for several months. Moreover, some other guests have received outside grants, for example the institute hosted a guest researcher from Korea as well. The number of foreign visitors of the institute – not counting the approximately 200 conference and summer school participants, neither the foreign employees – was near to 80 in 2012.

IV. Brief summary of national and international research proposals, winning in 2012

National grants

The research teams of the institute, compared to the availabilities, were exceptionally successful in 2012 with Hungarian OTKA (Hungarian Scientific Research Fund) project proposals. In 2012 further 4 research and 1 large scale research projects won support. The overall OTKA project support of the institute has increased slightly, and they managed to maintain that the faculty of the institute – with very few exceptions – are members of at least one running OTKA project. The relatively larger nominal increase in OTKA funds in 2012 compared to 2011 is partly explained by the fact that in case of four different OTKA projects, for different reasons, the 2011 yearly funding was transferred by the OTKA office only in 2012.

The projects for the "Momentum" call for special projects of the Hungarian Academy of Sciences remain very important and valuable to the institute. After the award in 2009 and final acceptance in 2012 of the cryptography project and the award of the second Momentum project in 2010 to study low dimensional topology one more young researcher was awarded a Momentum project to establish a research group in Group, graph and ergodic theory. Altogether these three projects gave the majority of the institute's non-OTKA national grant funds.

Apart from the above grants there were absolutely no other project proposal possibilities during 2012 in Hungary for theoretical or applied research in mathematics.

International grants

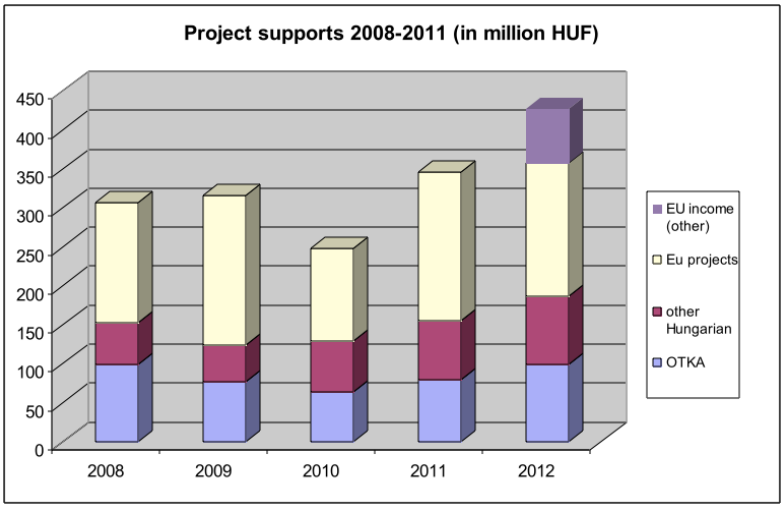
In FP7 new types of research projects, the Starting Independent Researcher and Advanced Investigators Grants of the European Research Council, namely, were introduced. These projects basically fund with several million Euros the research of small research groups run by the principal investigator (the starting independent researcher or the advanced investigator). There are a low number of funded projects and therefore these are highly competitive research calls. It is a great success for the institute that the PRIMEGAPS project, won in 2008, the DISCONV project, won in 2010 and the LTDBud project, won in 2011 were followed by another successful Advanced Investigators Grant application entitled "Regularity and Irregularity in Combinatorics and Number Theory" – headed by an Abel-prize recipient member of the institute and including several other members – in the 2012 round. The supported research will start in Spring 2013, after signing the research grant agreement, topping up the number of ERC Advanced Investigator Grants won by the institute to four.

Further four individual mobility project proposals were submitted in FP7 by foreign or Hungarian researchers residing in foreign countries, out of which one will be funded and start in 2013. Another two ERC Starting Grant project proposals were submitted, one advancing to the second round with the final decision made only in 2013.

Altogether, despite the constantly low level national project proposal opportunities the total research grant income of the institute in 2012 exceeded the grant income of 2011 (and all earlier years). The support of OTKA and Momentum projects were slightly more than the similar support of the previous year, while the support of international projects from the EU accounted for 2012 were slightly lower than those of 2011. A new element of the income scheme is the contractual income of the RTD project funded by the EU, but accounted at the institute as sales revenue, significantly contributing to the total, record income.

The running Momentum, OTKA and EU projects together with the Momentum project awarded in 2012 and the new ERC project starting in 2013 will jointly ensure that there will be no significant decrease in the total research grant income of the institute in 2013, despite the fact that the first Momentum and ERC Advanced grant projects ended in 2012 and will end in the first half of 2013, respectively.

The following diagram shows the amount of project support received during the last 5 years.



V. List of important publications in 2012

1. Abért M, Nikolov N: Rank gradient, cost of groups and the rank versus Heegaard genus problem. J Eur Math Soc, 14 (5): 1657-1677 (2012)

2. Ánh PN, Márki L, Vámos P: Divisibility theory in commutative rings: Bezout monoids. T Am Math Soc, 364 (8): 3967-3992 (2012)

3. Balka R, Buczolich Z, Elekes M: Topological Hausdorff dimension and level sets of generic continuous functions on fractals. Chaos Soliton Fract, 45 (12): 1579-1589 (2012)

4. Benyamini Y, Kroó A, Pinkus A: L^1-approximation and finding solutions with small support. Constr Approx, 36 (3): 399-431 (2012)

5. Berkes I, Horváth L, Rice G: Weak invariance principles for sums of dependent random functions. Stoch Proc Appl, 123 (2): 385-403 (2012)

6. Blomer V, Harcos G: A hybrid asymptotic formula for the second moment of Rankin-Selberg L-functions. P Lond Math Soc, 105 (3): 475-505 (2012)

7. Borgs C, Chayes JT, Lovász L, Sós VT, Vesztergombi K: Convergent sequences of dense graphs II. Multiway cuts and statistical physics. Ann Math, 176 (1): 151-219 (2012)

8. Böröczky K Jr, Lutwak E, Yang D, Zhang G: The log-Brunn-Minkowski inequality. Adv Math 231 (3-4): 1974-1997 (2012)

9. Csirmaz L, Tardos G: On-line secret sharing. Design Code Cryptogr, 63: 127-147 (2012)

10. Elek G, Szegedy B: A measure-theoretic approach to the theory of dense hypergraphs. Adv Math, 231 (3-4): 1731-1772 (2012)

11. Frankl P, Füredi Z: A new short proof of the EKR theorem. J Comb Theory A, 119 (6): 1388-1390 (2012)

12. Juhász I, Magidor M: On the maximal resolvability of monotonically normal spaces. Isr J Math, 102 (2): 637-666 (2012)

13. Miklós I, Zádori Z: Positive evolutionary selection of an HD motif on alzheimer precursor protein orthologues suggests a functional role. Plos Comput Biol, 8 (2): e1002356 (2012)

14. Némethi A: The cohomology of line bundles of splice-quotient singularities. Adv Math, 229 (4): 2503-2524 (2012)

15. Ozsváth A, Stipsicz A, Szabó Z: Combinatorial Heegaard Floer homology and nice Heegaard diagrams. Adv Math, 231 (1): 102-171 (2012)

16. Némethi A, Szilárd Á: Milnor fiber boundary of a non-isolated surface singularity. Berlin; Heidelberg: Springer Verlag, 2012. 240. (Lecture Notes in Mathematics; 2037.)

17. Andréka H, Ferenczi M, Németi I (Eds.): Cílindric-like algebras and algebraic logic. Berlin: Springer Verlag, 2012.